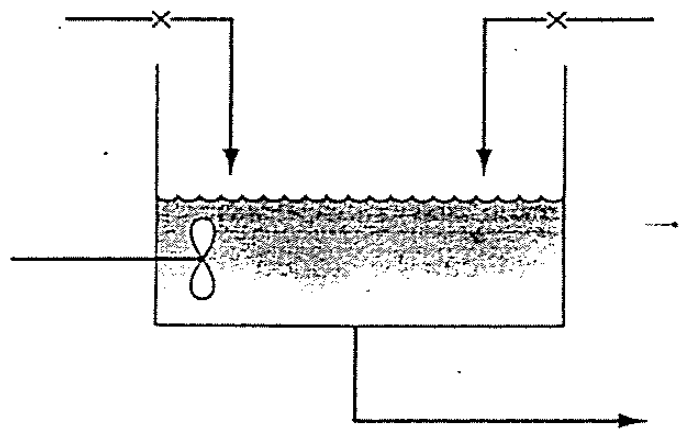
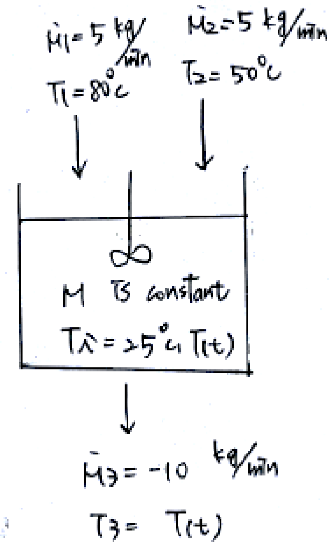
3.13 The mixing tank shown here initially contains 50 kg of water at 25 °C. Suddenly the two inlet valves and the single outlet valve are opened, so that two water streams, each with a flow rate of 5 kg/min, flow into the tank, and a single exit stream with a flow rate of 10 kg/min leaves the tank. The temperature of one inlet stream is 80°C, and that of the other is 50°C. The tank is well mixed, so that the temperature of the outlet stream is always the same as the temperature of the water in the tank.

a. Compute the steady-state temperature that will finally be obtained in the tank.

b. Develop an expression for the temperature of the fluid in the tank at any time.



3.13



(a)

<sol> Take tank as the system (open steady-state system)

no heat, no shaft work, no PV work

$$\frac{dU}{dt} \overset{0}{=} \dot{M}_1 \hat{H}_1 + \dot{M}_2 \hat{H}_2 + \dot{M}_3 \hat{H}_3 + \cancel{\dot{Q}}^{0} + \cancel{\dot{W}}^{0}$$

$$\Rightarrow 5\hat{H}_1 + 5\hat{H}_2 - 10\hat{H}_3 = 0$$

kg/min kg/min kg/min

$\hat{H}(T) = \hat{H}(T_R) + \int_{T_R}^{T} C_p(T)\,dT$, assume C_p is not a function of temp.

(T_R: reference temp)

$$= \hat{H}(T_R) + C_p(T - T_R)$$

$$\Rightarrow 5\hat{H}(T_R) + 5C_p(80 - T_R) + 5\hat{H}(T_R) + 5C_p(50 - T_R) - 10\hat{H}(T_R) - 10C_p(T_3 - T_R) = 0$$

$$\Rightarrow T_3 = 65°C$$

②

(b) Open Unsteady-state system

$$\frac{d\hat{U}^{\text{system}}}{dt} = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \dot{M}_3\hat{H}_3 + \cancel{Q}^{\,0} + \cancel{W}^{\,0}$$

$$\frac{d(M\hat{U})}{dt} = \frac{M\,d(\hat{U})}{dt} = M C_v \frac{dT}{dt}\;{}^{\prime\prime T_3}$$

$$\Rightarrow 50\,C_v\frac{dT_3}{dt} = 5\cancel{\hat{H}(T_R)} + 5\,C_p(80-T_R) + 5\cancel{\hat{H}(T_R)} + 5C_p(50-T_R)$$
$$-10\cancel{\hat{H}(T_R)} - 10C_p(T_3-T_R)$$

$$\Rightarrow 50\,C_v\frac{dT_3}{dt} = 5\,C_p(130) - 10\,C_p T_3$$

for liquid & solid: $\hat{H} = \hat{U} + p\hat{V}$

$\hat{V}$ is small for liq. & solid

$$\Rightarrow d\hat{H} = d\hat{U} + p\,d\hat{V} + \hat{V}\,dp$$

$\hat{V}$ remain constant for liq. & solid

$$\Rightarrow C_p\,dT \approx C_v\,dT$$

$$\Rightarrow C_p \approx C_v$$

$$\Rightarrow 10\frac{dT_3}{dt} = 130 - 2T_3$$

$$\Rightarrow \frac{dT_3}{dt} + \frac{1}{5}T_3 = 13$$

$$\Rightarrow T_3 = C_1\cdot e^{-\frac{1}{5}t} + C_2$$

at $t=0$. $T_3 = 25^\circ C$

at $t \to \infty$, $T_3 = 65^\circ C$

$$\Rightarrow \begin{cases} 25 = C_1 + C_2 \\ 65 = 0 + C_2 \end{cases} \Rightarrow C_1 = -40$$

$$\Rightarrow T_3 = -40^\circ C \cdot e^{-\frac{t}{5}} + 65^\circ C$$