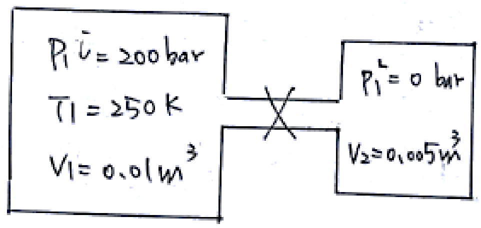
3.25 A .0.01-m^3 cylinder containing nitrogen gas initially at a pressure of 200 bar and 250 K is connected to another cylinder 0.005 m^3 in volume, which is initially evacuated. A valve between the two cylinders is opened until the pressures in the cylinders equalize. Find the final temperature and pressure in each cylinder if there is no heat flow into or out of the cylinder. You may assume that there is no heat transfer between the gas and the cylinder walls and that the gas is ideal with a constant-pressure heat capacity of 30 J/(mol K).

3.25



Find P_1^f, T_1^f, P_2^f, T_2^f

for ideal gas system.

$C_P^* = 30$ J/mol·K

$Q = 0$

<sol> From mass balance

$$N_1^i + N_2^i = N_1^f + N_2^f$$

$$\Rightarrow \frac{P_1^i V_1}{RT_1^i} + 0 = \frac{P_1^f V_1}{RT_1^f} + \frac{P_2^f V_2}{RT_2^f} \quad \because V_1 = 2V_2$$

$$\Rightarrow \frac{2P_1^i}{T_1^i} = \frac{2P_1^f}{T_1^f} + \frac{P_2^f}{T_2^f} \quad — ①$$

From energy balance

$$N_1^i \underline{U}_1^i + N_2^i \underline{U}_2^i = N_1^f \underline{U}_1^f + N_2^f \underline{U}_2^f \quad — ②$$

for ideal gas $\quad \underline{U}^{IG}_{(T)} = C_V^* T - C_P^* T_R \quad — ③$

substitute ③ into ②

$$\Rightarrow \frac{P_1^i \cancel{V_1}^{2}}{\cancel{R} T_1^i}(C_V^* T_1 - C_P^* T_R) + 0 = \frac{P_1^f \cancel{V_1}}{\cancel{R} T_1^f}(C_V^* T_1^f - C_P^* T_R)$$

$$+ \frac{P_2^f \cancel{V_2}^{1}}{\cancel{R} T_2^f}(C_V^* T_2^f - C_P^* T_R)$$

$$\Rightarrow \left(\left(\underbrace{\frac{-2P_1^i}{T_1^i} + \frac{2P_1^f}{T_1^f} + \frac{P_2^f}{T_2^f}}_{\overset{\|}{0}\ \text{from eq'n ⓪}}\right) C_p^* T_R + (2P_1^i - 2P_1^f - 2P_2^f) C_v^* = 0$$

since $C_v^* = 0$

$$\Rightarrow 2P_1^i = 2P_1^f + P_2^f$$

since $P_1^f = P_2^f$

$$\Rightarrow P_1^f = \frac{2}{3}P_1^i = P_2^f = \frac{2}{3} \times 200\ \text{bar} = 133\ \text{bar}$$

\#

substitute into eq'n ①

$$\frac{2 \cdot 200\ \text{bar}}{250\ \text{K}} = \frac{2 \cdot 133\ \text{bar}}{T_1^f\ \text{K}} + \frac{133\ \text{bar}}{T_2^f\ \text{K}}$$

$$\Rightarrow \frac{400}{250} = \frac{266}{T_1^f} + \frac{133}{T_2^f} \quad — ④$$

Take cylinder 1 as the system

From mass balance

$$\frac{dN_1}{dt} = \dot{N} \quad — ⑤$$

From energy balance (neglect $\frac{1}{2}Mv^2$, $M\Psi$)

substitute

$$\frac{d(N_1\underline{U}_1)}{dt} = \dot{N}\underline{H}_1 + \cancelto{0}{\dot{Q}} + \cancelto{0}{\dot{W}}$$

$$\Rightarrow \quad N_1 \cdot \frac{d\underline{U}_1}{dt} + \underline{U}_1 \frac{dN_1}{dt} = \frac{dN_1}{dt} \cdot \underline{H}_1$$

$$\Rightarrow \quad N_1 \cdot \frac{d\underline{U}_1}{dt} = (\underline{H}_1 - \underline{U}_1)\frac{dN_1}{dt}$$

From previous eq'n: $\underline{U}^{IG} = Cv^* T - Cp^* T_R$, $\frac{PV}{RT} = N$

$\underline{H}^{IG} = Cp^*(T - T_R)$

not function of t

$$\Rightarrow \quad \frac{P_1 \cancel{V_1}}{\cancel{R}T_1} \cdot \frac{d(Cv^* T_1 - \cancel{Cp^* T_R})}{dt} = (Cp^* T_1 - \cancel{Cp^* T_R} - Cv^* T_1 + \cancel{Cp^* T_R})\frac{d(\frac{P_1 \cancel{V_1}}{\cancel{R}T_1})}{dt}$$

V & R are constants

$$\Rightarrow \quad \frac{P_1}{T_1} Cv^* \frac{dT_1}{dt} = RT_1 \cdot \frac{d\frac{P_1}{T_1}}{dt}$$

$$\Rightarrow \quad \frac{Cv^*}{R} \cdot \frac{1}{T_1} \cdot \frac{dT_1}{dt} = \frac{T_1}{P} \cdot \frac{d(\frac{P_1}{T_1})}{dt}$$

$$\Rightarrow \quad \left(\frac{T_1^f}{T_1^i}\right)^{\frac{Cp^*}{R}} = \frac{P_1^f}{P_1^i} \quad \Rightarrow \quad \left(\frac{T_1^f}{250}\right)^{\frac{30}{8.31}} = \frac{133}{200}$$

$$\Rightarrow \quad T_1^f = 250 \cdot (0.665)^{0.277} = 223\,K \quad \#$$

substitute into eq'n ④

$$\frac{400}{250} = \frac{266}{223} + \frac{133}{T_2^f} \quad \Rightarrow \quad T_2^f = \frac{133}{0.41} = 324\,K \quad \#$$