4.11 Steam is produced at 70 bar and some unknown temperature. A small amount of stem is bled off just before entering a turbine and goes through an adiabatic throttling valve to atmospheric pressure. The temperature of the steam exiting the throttling valve is 400°C. The unthrottled steam is fed into the turbine, where it is adiabatically expanded to atmospheric pressure.

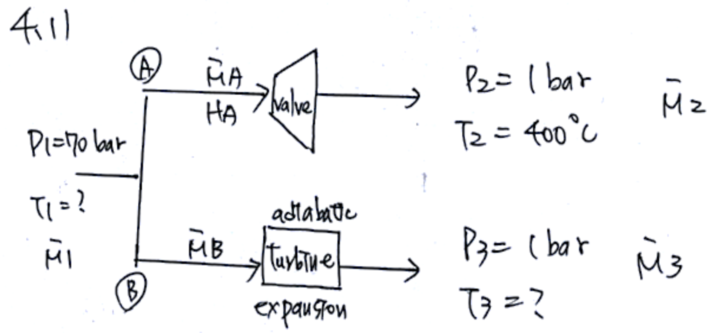
a. What is the temperature of the steam entering the turbine?

b. What is the maximum work per kilogram of steam that can be obtained using the turbine in its present mode of operation?

c. Tests on the turbine exhaust indicate that the steam leaving is a saturated vapor. What is the efficiency of the turbine and the entropy generated per kilogram of steam?

d. If the ambient temperature is 25℃ and the ambient pressure is 1 bar. what is the maximum possible work that could be obtained per kilogram of steam in any continuous process?

4.11



< Sol >

(a) From mass balance

$$\frac{dM}{dt} = \dot{M}_1 + \dot{M}_2 + \dot{M}_3 = 0 \Rightarrow \dot{M}_1 = -\dot{M}_2 - \dot{M}_3$$

Take valve as the system

$$\dot{M}_A = -\dot{M}_2 \quad \text{— ①}$$

Take turbine as the system

$$\dot{M}_B = -\dot{M}_3 \quad \text{— ②}$$

From energy balance

Take valve as the system

$$\frac{dU}{dt} = \dot{M}_A \hat{H}_A + \dot{M}_2 \hat{H}_2 + \frac{dQ}{dt} + \dot{W}_S - P\frac{dV}{dt} = 0$$

($\frac{dU}{dt} \to 0$ for steady state; $\frac{dQ}{dt} = 0$ adiabatic; $\dot{W}_S = 0$ No shaft work; $\frac{dV}{dt} = 0$)

$$\Rightarrow \dot{M}_A \hat{H}_A = -\dot{M}_2 \hat{H}_2$$

$$\because \dot{M}_A = -\dot{M}_2 \quad \text{— eq'n ①}$$

$\Rightarrow \hat{H}_A = \hat{H}_2$

from Mollier Diagram. $\hat{H}_2$ (1 bar, 400°C) = 3278 kJ/kg
$= \hat{H}_A$ (40 bar, T_1)

$\Rightarrow T_1 \approx 447°C$ #

(b) Take turbine as the system
From energy balance

$$\frac{dU}{dt} \overset{0\ ss}{=} \dot{M}_B \hat{H}_B + \dot{M}_3 \hat{H}_3 + \dot{Q} \overset{0\ adi}{} + \dot{W}$$

from eq'n ② $\dot{M}_B = -\dot{M}_3$

$\Rightarrow \dot{W} = \dot{M}_3 \hat{H}_B - \dot{M}_3 \hat{H}_3$

$\because \hat{H}_A = \hat{H}_1 = \hat{H}_2$

$= -\dot{M}_B (3278 \text{ kJ/kg} - \hat{H}_3)$ —③ ← $\hat{H}_3$ is unknown

From entropy balance

$$\frac{dS}{dt} \overset{0\ ss.}{=} \dot{M}_B \hat{S}_B + \dot{M}_3 \hat{S}_3 + \frac{\dot{Q}}{T} \overset{0\ adiabatic}{} + \dot{S}_{gen} \quad —④$$

for finding max. work ⇒ reversible process ⇒ $\dot{S}_{gen} = 0$

$\Rightarrow 0 = \dot{M}_B \hat{S}_B + \dot{M}_3 \hat{S}_3 \Rightarrow \hat{S}_B = \hat{S}_3$

$\because \hat{S}_B \doteq 6.62$ kJ/kg. from steam table

$\Rightarrow \hat{S}_3 = 6.62$ kJ/kg.K.

from P. 921 1 bar = 0.1 MPa

$\hat{S}_3 = 6.62 \text{ kJ/kg·K} < 7.3594 \text{ kJ/kg·K}$ (Saturated temp)

⇒ steam 3 consists of liquid & vapor

Let x = vapor fraction

$\hat{S}_3 = x \hat{S}_{vapor} + (1-x)\hat{S}_{liq.}$

from P. 919. $\hat{S}_{vapor} = 7.3594$, $\hat{S}_{liq.} = 1.3026$

⇒ $6.62 = x \cdot 7.3594 + (1-x) \times 1.3026$

⇒ $x \doteqdot 0.88$

⇒ $\hat{H}_3 = 0.88 \hat{H}_{vap} + 0.12 \hat{H}_{liq.}$

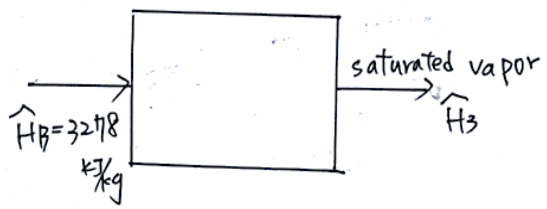
$= 0.88(2675.5) + 0.12(417.46) \text{ kJ/kg}.$

$\doteqdot 2405 \text{ kJ/kg}.$

from eq'n ③

$$-\frac{\dot{W}^{rev}}{\dot{M}_R} = 3278 - 2405 \text{ kJ/kg} = 873 \text{ kJ/kg}.$$

(c)



P.919 steam table

for saturated vapor. $\hat{H}_3 = 2675.5$ kJ/kg

$$\Rightarrow \frac{\dot{W}}{-\dot{M}_B} = 3278 - 2675.5 = 602.5 \text{ kJ/kg}$$

$$\eta = \frac{-\frac{\dot{W}}{\dot{M}_B}}{-\frac{\dot{W}^{rev}}{\dot{M}_B}} = \frac{602.5}{873} = 0.69 \%$$

from eq'n ④

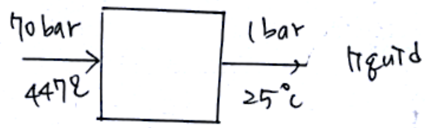
$$\dot{M}_B \hat{S}_B + \dot{M}_3 \hat{S}_3 + \dot{S}_{gen} = 0$$

$$\Rightarrow \dot{S}_{gen} = -\dot{M}_B \hat{S}_B - \dot{M}_3 \hat{S}_3$$

$$\Rightarrow \frac{\dot{S}_{gen}}{\dot{M}_B} = -\hat{S}_B + \hat{S}_3 = -6.62 + 7.3594$$

$$\fallingdotseq 0.74 \text{ kJ/kg}$$

(d)



From energy balance

$$\frac{dU}{dt} \overset{=0 \text{ s.s}}{=} \dot{M}_B \hat{H}_B + \dot{M}_3 \hat{H}_3 + \dot{Q} + \dot{W}$$

$$\Rightarrow \frac{\dot{W}}{-\dot{M}_B} = \hat{H}_B - \hat{H}_3 + \frac{\dot{Q}}{\dot{M}_B} \quad — (5)$$

From entropy balance

$$\frac{dS}{dt} = \dot{M}_B \hat{S}_B + \dot{M}_3 \hat{S}_3 + \frac{\dot{Q}}{T} + \dot{S}_{gen} \overset{\text{"}0}{}$$

for max work system operates reversible

$$\Rightarrow \frac{\dot{Q}}{\dot{M}_B} = (\hat{S}_3 - \hat{S}_B) T \quad — (6) \qquad T = 25°C$$

∵ all heat transfer at ambient temp.

(6) substitute into (5)

$$\Rightarrow \frac{\dot{W}}{-\dot{M}_B} = (\hat{H}_B - T\hat{S}_B) - (\hat{H}_3 - T\hat{S}_3)$$

P917

$$= (3278 - 298 \times 6.62) - (104.89 - 298 \times 0.3674)$$

$$\doteqdot 1309.8 \ kJ/kg.$$