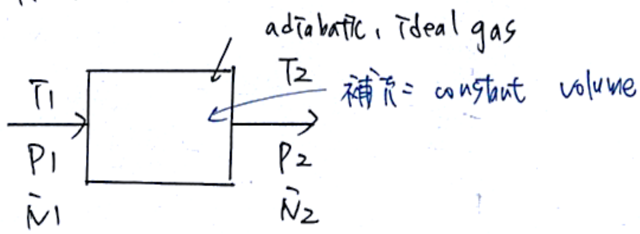
4.24 An adiabatic turbine is operating with an ideal gas working fluid of fixed inlet temperature and pressure, T_1 and P_1, respectively, and a fixed exit pressure, P_2.

Show that

a. The minimum outlet temperature, T_2, occurs when the turbine operates reversibly, that is, when $S_{gen}=0$.

b. The maximum work that can be extracted from the turbine is obtained when $S_{gen}=0$.

4.24



a1. Show minimum T_2 occurs when $\dot{S}_{gen}=0$

From mass balance

$$\frac{dN}{dt} = \dot{N}_1 + \dot{N}_2 = 0 \Rightarrow \dot{N}_2 = -\dot{N}_1 \quad — ①$$

From energy balance

adiabatic   constant vol.

$$\overbrace{\frac{dU}{dt}}^{=0\ SS} = \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \cancel{\dot{Q}}^{\,0} + \dot{W}_s - P\overbrace{\frac{dV}{dt}}^{0}$$

$$\Rightarrow \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \dot{W}_s = 0 \quad — ②$$

① substitute into ②

$$\Rightarrow \dot{N}_1 \underline{H}_1 - \dot{N}_1 \underline{H}_2 + \dot{W}_s = 0$$

$\because$ ideal gas $\underline{H} = C_p^* T$

$$\Rightarrow \dot{W}_s = \dot{N}_1 C_p^* (T_2 - T_1)$$

From entropy balance

adiabatic

$$\overbrace{\frac{dS}{dt}}^{=0\ SS} = \dot{N}_1 \underline{S}_1 + \dot{N}_2 \underline{S}_2 + \frac{\cancel{\dot{Q}}^{\,0}}{T} + \dot{S}_{gen}$$

$$\Rightarrow \dot{S}_{gen} = \dot{N}_1 (\underline{S}_2 - \underline{S}_1) \quad — ③$$

$\because d\underline{H} = Td\underline{S} + \underline{V}dP$ $\quad \underline{V} = \frac{RT}{P}$ Ideal gas

$\Rightarrow d\underline{S} = \frac{d\underline{H}}{T} - \frac{\underline{V}dP}{T}$

$\Rightarrow \Delta\underline{S} = Cp^* \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$ — (4)

(4) substitute into (3)

$\Rightarrow \dot{S}gen = \dot{N}_1 (Cp^* \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1})$

$\Rightarrow \dfrac{\frac{\dot{S}gen}{\dot{N}_1} + R\ln\frac{P_2}{P_1}}{Cp^*} = \ln\frac{T_2}{T_1}$

$\Rightarrow T_2 = T_1 \cdot \exp\left[\dfrac{\frac{\dot{S}gen}{\dot{N}_1} + R\ln\frac{P_2}{P_1}}{Cp^*}\right]$

$\because \dot{S}gen \geq 0$

$\Rightarrow$ minimum T_2 occurs when $\dot{S}gen = 0$

$\dot{W}s = \dot{N}_1 Cp^* (T_2 - T_1)$. since $T_1 > T_2$

$\Rightarrow$ minimum $T_2 \Rightarrow$ minimum work

$\Rightarrow$ maximum work can be extracted.

when $\dot{S}gen = 0$.