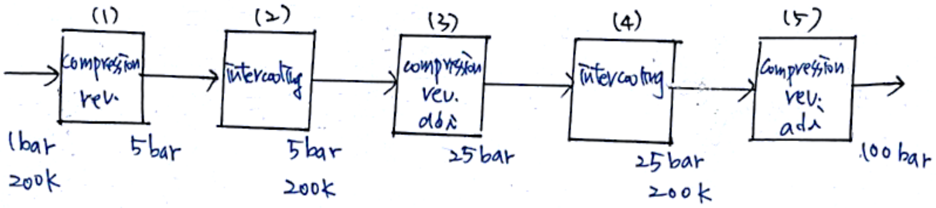
5.8 As in Illustration 5.1-1 it is desired to produce liquefied methane; however, the conditions are now changed so that the gas is initially available at 1 bar and 200 K,
and methane leaving the cooler will be at 100 bar and 200 K. The flash drum is adiabatic and operates at 1 bar, and each compressor stage can be assumed to operate reversibly and adiabatically. A three-stage compressor will be used, with the first stage compressing the gas from 1 bar to 5 bar, the second stage from 5 bar to 25 bar, and the third stage from 25 bar to 100 bar. Between stages the gas will be isobarically cooled to 200 K.

a. Calculate the amount of work required for each kilogram of methane that passes through the compressor in the simple liquefaction process.
b. Calculate the fractions of vapor and liquid leaving the flash drum in the simple liquefaction process of Fig. 5.1-1 and the amount of compressor work required for each kilogram of LNG produced.
c. Assuming that the recycled methane leaving the heat exchanger in the Linde process (Fig. 5.1-2) is at 1 bar and 200 K, calculate the amount of compressor work required per kilogram of LNG produced.

5.8

(a)



Take (1) as the system

Mass Balance

$$\cancel{\frac{dM}{dt}}^{\,0\ s.s.} = \dot{M}_{in} + \dot{M}_{out} \Rightarrow \dot{M}_{out} = -\dot{M}_{in}$$

Energy Balance

$$\frac{dU}{dt} = \dot{M}_{in}\hat{H}_{in} + \dot{M}_{out}\hat{H}_{out} + \dot{Q} + \dot{W}$$

$$\Rightarrow \dot{W} = \dot{M}_{in}(\hat{H}_{out} - \hat{H}_{in}) \Rightarrow \frac{\dot{W}}{\dot{M}_{in}} = \hat{H}_{out} - \hat{H}_{in}$$

Entropy Balance

$$\cancel{\frac{dS}{dt}}^{\,0\ s.s.} = \dot{M}_{in}\hat{S}_{in} + \overset{\overset{-\dot{M}_{in}}{\parallel}}{\boxed{\dot{M}_{out}}}\,\hat{S}_{out} + \cancel{\frac{\dot{Q}}{T}}^{\,adi} + \cancel{\dot{S}_{gen}}^{\,rev.}$$

$$\Rightarrow \hat{S}_{in} = \hat{S}_{out}$$

From Fig. 7.3-2: $\hat{H}_{in}(200k, 1bar) = 767\ KJ/kg$

$\hat{S}_{in}(200k, 1bar) = 6.5\ KJ/kg.k = \hat{S}_{out}$

$\Rightarrow \hat{H}_{out}(S = 6.5\ KJ/kg.k,\ 5bar) = 963\ KJ$

$$\Rightarrow \frac{\dot{W}}{\dot{M}_{in}} = 963 - 767 = 196\ KJ/kg$$

Take (2) as the system

$\frac{\dot{W}_{in}}{\dot{M}_{in}} = \hat{H}_{out} - \hat{H}_{in}$

From Fig 3.3-2: $\hat{H}_{in}$ (200k, 5 bar)
$= 760$ KJ/kg.

$\hat{S}_{in}$(200k, 5 bar)
$= 5.65$ KJ/kg.k

$\Rightarrow \hat{H}_{out}$ ($S = 5.65$ KJ/kg.k , 25bar)
$= 960$ KJ/kg

$\Rightarrow \frac{\dot{W}_{in}}{\dot{M}_{in}} = 960 - 760 = 200$ KJ/kg.k.

Take (3) at the system

$\frac{\dot{W}_{in}}{\dot{M}_{in}} = \hat{H}_{out} - \hat{H}_{in}$.

From Fig 3.3.2: $\hat{H}_{in}$ (200k, 25 bar) $= 718$ KJ/kg.

$\hat{S}_{in}$ (200k, 25 bar) $= 4.65$ KJ/kg.k

$\Rightarrow \hat{H}_{out}$ ($\hat{S} = 4.65$ KJ/kg.k , 100bar) $= 855$ KJ/kg

$\Rightarrow \frac{\dot{W}_{in}}{\dot{M}_{in}} = 855 - 718 = 137$ KJ/kg

The total Work:

$196 + 200 + 137 = 533$ KJ/kg.

(b) Take the value as the system.

From energy balance

$$\frac{d\cancelto{0\ ss}{U}}{dt} = \dot{M}_{in}\hat{H}_{in} + \overset{= -\dot{M}_{in}}{\dot{M}_{out}}\hat{H}_{out} + \cancelto{adi}{\dot{Q}} + \cancelto{\text{no work.}}{\dot{W}}$$

$$\Rightarrow \hat{H}_{in} = \hat{H}_{out}$$

mass fraction of Liq.

Fig 3.3-2 $\hat{H}_{in}(200K, 100\ bar) = (1-x)\hat{H}(sat. Liq. 1\ bar)$

$+ x\hat{H}(sat. vap. 1\ bar)$

$$\Rightarrow 423 = (1-x)\cdot 71 + x \cdot 582$$

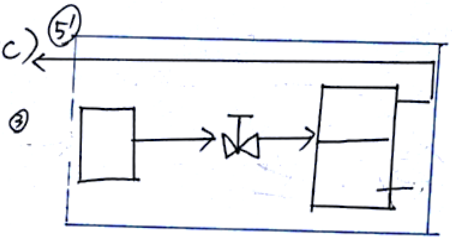
From Fig 3.3-2

$$\Rightarrow x = 0.689$$

$\Rightarrow \frac{533}{(1-0.689)} = 1713$ kJ of work are required

for each kg. of LNG produced.

(c)



From mass balance

$$\frac{dM}{dt} = \dot{M}_3 + \dot{M}_5' + M_6 \Rightarrow \dot{M}_6 + \dot{M}_5' = -\dot{M}_3$$

From energy balance

$$\overset{\text{s.s.}}{\cancel{\frac{dU}{dt}}} = \dot{M}_3 \hat{H}_3 + \dot{M}_5' \hat{H}_5' + \dot{M}_6 \hat{H}_6 + \overset{\text{adi}}{\cancel{\dot{Q}}} + \overset{\text{no work.}}{\cancel{\dot{W}}}$$

$$\Rightarrow \dot{M}_3 \hat{H}_3 + \dot{M}_5' \hat{H}_5' + \dot{M}_6 \hat{H}_6 = 0$$

Let $\dot{M}_3 = 1$ $\dot{M}_5' = -w.$ $\dot{M}_6 = -1 + w$

$$\underset{\downarrow\ 423\ \text{kJ/kg}}{\hat{H}_3\ (200\text{k},\ 100\ \text{bar})} - w \underset{\downarrow\ 718\ \text{kJ/kg}}{\hat{H}_5'\ (\text{sat. vap. } 1\text{bar})}$$

$$+ (-1 + w)\underset{\downarrow\ 71\ \text{kJ/kg}}{\hat{H}_6\ (\text{sat liq. } 1\ \text{bar})} = 0$$

$$\Rightarrow w = 0.544 \qquad 1 - w = 0.456$$

$$\frac{5.33}{0.456} = 1168\ \text{kJ/kg} \ \# \ \text{of LNG produced.}$$