6.52 .A fluid is described by the Clausius equation of state

$$P = \frac{RT}{\underline{V} - b}$$

where b is a constant. Also, the ideal gas heat capacity is given by

$$C_{\mathrm{P}}^{*} = \alpha + \beta T + \gamma T^{2}$$

For this fluid. obtain explicit expressions for

a. A line of constant enthalpy as a function of pressure and temperature

b. A line of constant entropy as a function of temperature and pressure

c. Does this fluid have a Joule-Thomson inversion temperature?

6.52

a. $\Delta \underline{H} = \int_{P_1\ T_1}^{P=0\ T_1} \left[\underline{V} - T\left(\frac{\partial \underline{V}}{\partial T}\right)_p\right] dp + \int_{P=0, T_1}^{P=0, T_2} C_p^* dT$

$+ \int_{P=0, T_2}^{P_2, T_2} \left[\underline{V} - T\left(\frac{\partial \underline{V}}{\partial T}\right)_p\right] dp$

$P = \frac{RT}{\underline{V}-b} \Rightarrow \underline{V} = \frac{RT}{P} + b \quad , \left(\frac{\partial \underline{V}}{\partial T}\right)_p = \frac{R}{P} , \quad \underline{V} - T\frac{R}{P} = b$

$\Rightarrow \Delta \underline{H} = \int_{P_1\ T_1}^{P=0\ T_1} b\ dp + \int_{P=0, T_1}^{P=0, T_2} C_p^* dT + \int_{P=0, T_2}^{P=P_2, T_2} b \cdot dp$

where $C_p^* = \alpha + \beta T + \gamma T^2$

$\Rightarrow \Delta \underline{H} = -bP_1 + \alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + bP_2$

$= 0$ for constant enthalpy

$\Rightarrow bP + \alpha T + \frac{\beta}{2}T^2 + \frac{\gamma}{3}T^3$ for line of constant enthalpy

b. $$\Delta S = \int_{P_1, T_1}^{P=0, T_1} \left(\frac{\partial V}{\partial T}\right)_p dp + \int_{P=0, T_1}^{P=0, T_2} \frac{C_p^*}{T} dT + \int_{P=0, T_2}^{P_2, T_2} \left(\frac{\partial V}{\partial T}\right)_p dp$$

$$= R \ln \frac{P_2}{P_1} + \alpha \ln \frac{T_2}{T_1} + \beta (T_2 - T_1) + \frac{\gamma}{2}(T_2^2 - T_1^2) = 0$$

$$\Rightarrow b \ln P + \alpha \ln T + \beta T + \frac{\gamma}{2} T^2 \text{ for line of constant } S$$

c. Joule-Thomson inversion temp.

$\Rightarrow$ the temperature at which $\left(\frac{\partial T}{\partial P}\right)_H = 0$

$$\left(\frac{\partial T}{\partial P}\right)_H \left(\frac{\partial P}{\partial H}\right)_T \left(\frac{\partial H}{\partial T}\right)_P = -1$$

$$\mu = \left(\frac{\partial T}{\partial P}\right)_H = \frac{-\left(\frac{\partial H}{\partial P}\right)_T}{\left(\frac{\partial H}{\partial T}\right)_P} = -\frac{\left[V - T\left(\frac{\partial V}{\partial T}\right)_P\right]}{C_P}$$

$$= -\frac{b}{C_P^*} = 0$$

$\because C_P^* \neq 0 \quad \therefore$ as $b = 0$. $\mu = 0$.