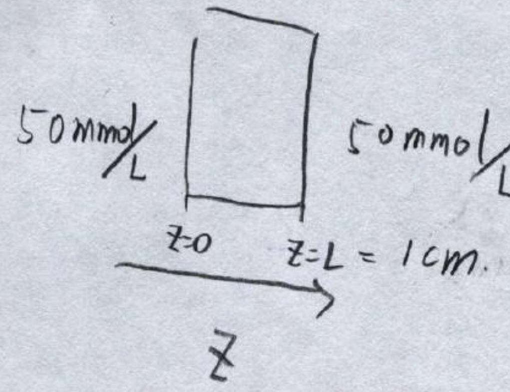
27.16



$$\begin{cases} t=0. & z \geq 0 & C_A(z, t=0) = C_{A0} \\ t>0. & z=0. & C_A(0,t) = C_{A \cdot S}. \\ t>0 & z=L. & C_A(L,t) = C_{A \cdot S} \end{cases}$$

at $t = 42$ hours. $z = \frac{L}{2} = 0.5\,cm$,

$C_A = 48.5$ mmol/L, find D_{AB} = ?

(Sol) From differential eq'n of mass transfer

$$\vec{\nabla} \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \cancel{R_A} = 0$$

No reaction

in z direction only

$$\frac{\partial N_{A \cdot z}}{\partial z} + \frac{\partial C_A}{\partial t} = 0 \quad \text{—①}$$

From Fick's eq'n.

No bulk motion

$$\vec{N}_A = -D_{AB} \vec{\nabla} C_A + \cancel{y_A (\vec{N}_A + \vec{N}_B)}$$

in z direction only

$$\Rightarrow N_{A \cdot z} = -D_{AB} \frac{\partial C_A}{\partial z} \quad \text{—②}$$

Substitute eq'n ② into eq'n ①

$$-D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial t} = 0$$

$$\Rightarrow \frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2} \quad \text{—③}$$

②

Let $Y = \frac{C_A - C_{AS}}{C_{A0} - C_{AS}}$, $\begin{cases} t=0 & Y=1 \\ z=0 & Y=0 \\ z=L & Y=0 \end{cases}$

eqn ③ $\frac{\partial Y}{\partial t} = D_{AB} \frac{\partial^2 Y}{\partial z^2}$

Let $Y = T_{(t)} Z_{(z)}$

$\Rightarrow \frac{1}{D_{AB}} \frac{1}{T} \frac{\partial T}{\partial t} = \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2}$

Let $\frac{1}{D_{AB}} \frac{1}{T} \frac{\partial T}{\partial t} = \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = -\lambda^2$

$\Rightarrow Y = T_{(t)} Z_{(z)} = \left[C_1' \cos(\lambda z) + C_2' \sin(\lambda z) \right] e^{-D_{AB}\lambda^2 t}$

at $z=0$, $Y=0$

$\Rightarrow 0 = C_1' e^{-D_{AB}\lambda^2 t} \Rightarrow C_1' = 0$

at $z=L$ $Y=0$

$\Rightarrow 0 = C_2' \sin(\lambda L) e^{-D_{AB}\lambda^2 t}$, $\because C_2' \neq 0$

$\therefore \lambda L = n\pi \Rightarrow \lambda = \frac{n\pi}{L}$, $n = 1, 2, 3 \ldots$

$\Rightarrow Y = C_2' \sum_{n=1}^{\infty} \sin\left(\frac{n\pi z}{L}\right) e^{-D_{AB}\left(\frac{n\pi}{L}\right)^2 t}$

at $t=0$, $Y=1$

$$\Rightarrow 1 = C_2' \sum_{n=1}^{\infty} \sin\left(\frac{n\pi z}{L}\right)$$

$$\Rightarrow \int_0^L \sin\frac{m\pi z}{L}\,dz = C_2' \int_0^L \sin\frac{n\pi z}{L}\sin\frac{m\pi z}{L}\,dz$$

$$= C_2' \cdot \frac{L}{2} \quad \text{as } n = m.$$

$$\Rightarrow C_2' = \frac{2}{L}\int_0^L \sin\frac{n\pi z}{L}\,dz = \begin{cases} \frac{4}{n\pi} & \text{when } n = 1, 3, 5 \ldots \\ 0 & \text{when } n = 0, 2, 4, 6 \ldots \end{cases}$$

不符.

$$\Rightarrow C_2' = \frac{4}{n\pi}, \quad n = 1, 3, 5 \ldots$$

$$\Rightarrow Y = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi z}{L}\,e^{-D_{AB}\left(\frac{n\pi}{L}\right)^2 t} = \frac{C_A - C_{AS}}{C_{A0} - C_{AS}}$$

$$\Rightarrow C_A = C_{AS} + (C_{A0} - C_{AS})\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi z}{L}\,e^{-D_{AB}\left(\frac{n\pi}{L}\right)^2 t}$$

$n = 1, 3, 5 \ldots$

$C_{AS} = 50$ mmol/L. $C_{A0} = 0$. $z = \frac{L}{2} = 0.5$ cm.

at $t = 42$ hours $C_A = 48.5$ mmol/L $\Rightarrow$ find D_{AB}