27.4

$C(0.05\,m, 259200\,sec)$ ①

〃

$C_A(5\,cm, 72\,hours) = ?$

$z = 0$

$z \downarrow \quad \downarrow C_6H_6\ (A)$

Solubility $= 24\ mol/m^3$.

stagnant

$D_{A \cdot B} = 10^{-9}\ m^2/s$

$z = 1\,m$

For semi-infinite diffusion.

From differential eqn of mass transfer

$$\vec{\nabla}\cdot\vec{N_A} + \frac{\partial C_A}{\partial t} - \cancel{R_A} = 0$$

° No reaction

(in z direction only)

$$\Rightarrow \vec{\nabla}\cdot\vec{N_A} + \frac{\partial C_A}{\partial t} = 0 \quad \Rightarrow \quad \frac{\partial N_{A \cdot z}}{\partial z} + \frac{\partial C_A}{\partial t} = 0 \quad -①$$

From Fick's eqn.

$$\vec{N_A} = -CD_{AB}\vec{\nabla} y_A + \cancel{y_A(\vec{N_A} + \vec{N_B})}$$

° stagnant film no bulk motion

$$\Rightarrow N_{A \cdot z} = -D_{AB}\frac{\partial C_A}{\partial z} \quad -② \text{ (in z direction only)}$$

eqn ② into eqn ①

$$\Rightarrow -D_{AB}\frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial t} = 0$$

$$\Rightarrow \frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

B.Cs & IC

$$\begin{cases} \text{at } t=0, & C_A(z,0)=0 \\ \text{at } t>0 \quad z=0. & C_A(0,t)=C_{AS}. \\ \text{at } t>0. \quad z\to\infty & C_A(\infty,t)=0 \end{cases}$$

Laplace transformation respect to time. $\bar{C}_A = \mathcal{L}\{C_A\}$

$$s\bar{C}_A - \underbrace{C_A(z,0)}_{=0} = D_{AB}\frac{\partial^2 \bar{C}_A}{\partial z^2}$$

$$\Rightarrow s\bar{C}_A = D_{AB}\frac{\partial^2 \bar{C}_A}{\partial z^2} \quad -③$$

B.Cs:

$$\begin{cases} \text{at } t>0. \quad z=0. & \bar{C}_A(0,t) = \mathcal{L}\{\overset{\text{constant}}{C_{AS}}\} = \frac{1}{s}C_{AS} & -\text{B.C①} \\ \text{at } t>0. \quad z\to\infty & \bar{C}_A(\infty,t) = \mathcal{L}\{0\} = 0 & -\text{B.C②} \end{cases}$$

eq'n ③ : $\frac{\partial^2 \bar{C}_A}{\partial z^2} - \frac{s}{D_{AB}}\bar{C}_A = 0$

$$\Rightarrow \bar{C}_A = A_1 e^{\sqrt{\frac{s}{D_{AB}}}z} + B_1 e^{-\sqrt{\frac{s}{D_{AB}}}\cdot z}$$

substitute B.C.②

$$0 = A_1 e^{\sqrt{\frac{s}{D_{AB}}}\infty} + \underbrace{B_1 e^{-\sqrt{\frac{s}{D_{AB}}}\infty}}_{=0}$$

$$\Rightarrow A_1 = 0$$

Substitute B.C ①

$$\frac{1}{s} C_{AS} = B_1 e^{-\sqrt{\frac{s}{D_{AB}}} \cdot 0} \quad =1$$

$$\Rightarrow B_1 = \frac{1}{s} C_{AS}$$

$$\Rightarrow \overline{C_A} = \frac{1}{s} C_{AS}\, e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z}$$

$$\Rightarrow C_A = L^{-1}\left\{\overline{C_A}\right\} = L^{-1}\left\{\frac{1}{s} C_{AS}\, e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z}\right\}$$

$$= C_{AS} \times L^{-1}\left\{\frac{1}{s} e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z}\right\}$$

$$= C_{AS}\, \mathrm{erfc}\left(\frac{z}{2\sqrt{D_{AB} \cdot t}}\right)$$

$$= C_{AS}\left[1 - \mathrm{erf}\left(\frac{z}{2\sqrt{D_{AB} \cdot t}}\right)\right]$$

$$= 24\ \mathrm{mol}/\mathrm{m}^3 \cdot \left[1 - \mathrm{erf}\left(\frac{0.05\ \mathrm{m}}{2\sqrt{10^{-9}\ \mathrm{m}^2/\mathrm{s} \cdot 259200\ \mathrm{s}}}\right)\right]$$

$$= 24\ \mathrm{mol}/\mathrm{m}^3 \cdot \left[1 - \mathrm{erf}(1.553)\right]$$

$$= 24\ \mathrm{mol}/\mathrm{m}^3\ (1 - 0.972)$$

$$= 0.672\ \mathrm{mol}/\mathrm{m}^3.$$