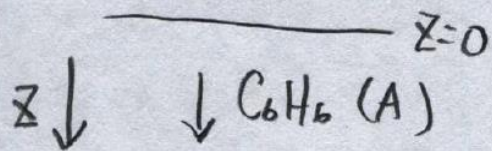


27.4

$C(0.05\text{ m}, > 900\text{ sec})$ ①

$C_A(5\text{ cm}, > \text{hours}) = ?$

Solubility = 24 mol/m^3 .



stagnant

$$D_{A.B} = 10^{-9}\text{ m}^2/\text{s}$$



For semi-infinite diffusion.

From differential eqn of mass transfer

$$\vec{\nabla} \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0 \quad \text{No reaction}$$

(in z direction only)

$$\Rightarrow \vec{\nabla} \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} = 0 \quad \Rightarrow \frac{\partial N_{A,z}}{\partial z} + \frac{\partial C_A}{\partial t} = 0 \quad \text{--- ①}$$

From Fick's eqn.

$$\vec{N}_A = -CD_{AB} \vec{\nabla} y_A + y_A (\vec{N}_A + \vec{N}_B) \quad \text{stagnant film, no bulk motion}$$

$$\Rightarrow N_{A,z} = -D_{AB} \frac{\partial C_A}{\partial z} \quad \text{--- ② (in z direction only)}$$

eqn ② into eqn ①

$$\Rightarrow -D_{AB} \frac{\partial^2 C_A}{\partial z^2} + \frac{\partial C_A}{\partial t} = 0$$

$$\Rightarrow \frac{\partial C_A}{\partial t} = D_{AB} \frac{\partial^2 C_A}{\partial z^2}$$

B.C.s & IC

(2)

$$\begin{cases} \text{at } t=0, C_A(z,0) = 0 \\ \text{at } t>0, z=0, C_A(0,t) = C_{AS} \\ \text{at } t>0, z \rightarrow \infty, C_A(\infty,t) = 0 \end{cases}$$

Laplace transformation respect to time. $\bar{C}_A = L\{C_A\}$

$$s\bar{C}_A - \underbrace{C_A(z,0)}_0 = D_{AB} \frac{\partial^2 \bar{C}_A}{\partial z^2}$$

$$\Rightarrow s\bar{C}_A = D_{AB} \frac{\partial^2 \bar{C}_A}{\partial z^2} \quad \text{--- (3)}$$

B.C.s =

$$\begin{cases} \text{at } t>0, z=0, \bar{C}_A(0,t) = L\{C_{AS}\} = \frac{1}{s} C_{AS} \quad \text{--- B.C. (1)} \\ \text{at } t>0, z \rightarrow \infty, \bar{C}_A(\infty,t) = L\{0\} = 0 \quad \text{--- B.C. (2)} \end{cases}$$

constant

eq'n (3): $\frac{\partial^2 \bar{C}_A}{\partial z^2} - \frac{s}{D_{AB}} \bar{C}_A = 0$

$$\Rightarrow \bar{C}_A = A_1 e^{\sqrt{\frac{s}{D_{AB}}} z} + B_1 e^{-\sqrt{\frac{s}{D_{AB}}} z}$$

substitute B.C. (2)

$$0 = A_1 e^{\sqrt{\frac{s}{D_{AB}}} \infty} + B_1 e^{-\sqrt{\frac{s}{D_{AB}}} \infty} = 0$$

$$\Rightarrow A_1 = 0$$

Substitute B.C ①

③

$$\frac{1}{s} C_{AS} = B_1 e^{-\sqrt{\frac{s}{D_{AB}}} \cdot 0} = 1$$

$$\Rightarrow B_1 = \frac{1}{s} C_{AS}$$

$$\Rightarrow \bar{C}_A = \frac{1}{s} C_{AS} e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z}$$

$$\Rightarrow C_A = L^{-1} \{ \bar{C}_A \} = L^{-1} \left\{ \frac{1}{s} C_{AS} e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z} \right\}$$

$$= C_{AS} \cdot L^{-1} \left\{ \frac{1}{s} e^{-\sqrt{\frac{s}{D_{AB}}} \cdot z} \right\}$$

$$= C_{AS} \operatorname{erfc} \left(\frac{z}{2\sqrt{D_{AB} \cdot t}} \right)$$

$$= C_{AS} \left[1 - \operatorname{erf} \left(\frac{z}{2\sqrt{D_{AB} \cdot t}} \right) \right]$$

$$= 24 \frac{\text{mol}}{\text{m}^3} \cdot \left[1 - \operatorname{erf} \left(\frac{0.05 \text{ m}}{2\sqrt{10 \frac{\text{m}^2}{\text{s}} \cdot 5900 \text{ s}}} \right) \right]$$

$$= 24 \frac{\text{mol}}{\text{m}^3} \cdot [1 - \operatorname{erf}(1.553)]$$

$$= 24 \frac{\text{mol}}{\text{m}^3} (1 - 0.972)$$

$$= 0.672 \frac{\text{mol}}{\text{m}^3}$$