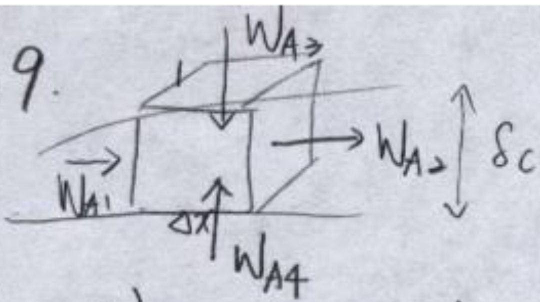
28.19.



①

$$W_{A1} + W_{A3} + W_{A4} = W_{A2}$$

$$\int_0^{\delta_c} C_A \cdot v_x \cdot 1 \cdot dy\Big|_x + C_\infty\left[\frac{\partial}{\partial x}\int_0^{\delta_c} v_x \cdot dy\right]\Delta x$$

$$+ k_c\left(C_{AS} - C_{A\infty}\right) 1 \cdot \Delta x = \int_0^{\delta_c} C_A \cdot v_x \cdot 1 \cdot dy\Big|_{x+\Delta x}$$

$\Rightarrow$ dividing Δx & Let $\Delta x \to 0$

$$\Rightarrow C_{A\infty}\left[\frac{d}{dx}\int_0^{\delta_c} v_x\, dy\right] + k_c\left(C_{AS} - C_{A\infty}\right) = \frac{d}{dx}\int_0^{\delta_c} C_A \cdot v_x\, dy$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_c}\left(C_A - C_{A\infty}\right) v_x\, dy = k_c\left(C_{AS} - C_{A\infty}\right) \quad — ①$$

Assume. $v_x = \alpha + \beta y^{1/7}$

at $\begin{cases} y = 0, & v_x = 0 \\ y = \delta, & v_x = v_\infty \end{cases} \Rightarrow \begin{cases} 0 = \alpha \\ v_\infty = \beta\,\delta^{1/7} \Rightarrow \beta = \dfrac{v_\infty}{\delta^{1/7}} \end{cases}$

Assume $C_A - C_{A\infty} = \eta + \xi y^{1/7}$ $\quad \Rightarrow v_x = v_\infty\left(\frac{y}{\delta}\right)^{1/7} \quad — ②$

at $\begin{cases} y = 0, & C_A = C_{AS} \\ y = \delta_c, & C_A = C_{A\infty} \end{cases} \Rightarrow \begin{cases} y = 0. \; C_A - C_{A\infty} = C_{AS} - C_{A\infty} = \eta + \xi \cdot 0 \\ \qquad \Rightarrow \eta = C_{AS} - C_{A\infty} \\ y = \delta_c, \; C_A - C_{A\infty} = C_{A\infty} - C_{A\infty} = 0 \\ \qquad = \eta + \xi (\delta_c)^{1/7} \end{cases}$

$$\Rightarrow \xi = -\frac{\eta}{(\delta_c)^{1/7}} = \frac{C_{A\infty} - C_{AS}}{(\delta_c)^{1/7}}$$

$$\Rightarrow C_A - C_{A\infty} = \left(C_{AS} - C_{A\infty}\right) + \frac{C_{A\infty} - C_{AS}}{(\delta_c)^{1/7}}\, y^{1/7}$$

$$\Rightarrow \frac{C_A - C_{A\infty}}{C_{AS} - C_{A\infty}} = 1 - \left(\frac{y}{\delta_c}\right)^{1/7} \quad -(3)$$

substitute

eq'n (2) & (3) into eq'n (1)

$$\frac{d}{dx}\left[\int_0^{\delta_c} (C_A - C_{A\infty})\, v_x\, dy\right] = k_c\,(C_{AS} - C_{A\infty})$$

$$\Rightarrow \frac{d}{dx}\left[\int_0^{\delta_c} \cancel{(C_{AS} - C_{A\infty})}\left(1 - \frac{y}{\delta_c}\right)^{1/7} \cdot v_\infty \left(\frac{y}{\delta}\right)^{1/7} dy\right] = k_c\,\cancel{(C_{AS} - C_{A\infty})}$$

$$\Rightarrow \frac{d}{dx}\left[\int_0^{\delta_c} \left(1 - \frac{y}{\delta_c}\right)^{1/7} \left(\frac{y}{\delta}\right)^{1/7} dy\right] = \frac{k_c}{v_\infty}$$

$$\Rightarrow \frac{d}{dx}\left[\int_0^{\delta_c} \left(\frac{1}{\delta^{1/7}}\, y^{1/7} - \frac{1}{\delta_c^{1/7}\delta^{1/7}}\, y^{2/7}\right) dy\right] = \frac{k_c}{v_\infty}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{7}{8}\frac{1}{\delta^{1/7}}\, y^{8/7} - \frac{7}{9}\frac{1}{\delta_c^{1/7}\delta^{1/7}}\, y^{9/7}\right)\Bigg|_0^{\delta_c} = \frac{k_c}{v_\infty}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{7}{8}\frac{1}{\delta^{1/7}}\, \delta_c^{8/7} - \frac{7}{9}\frac{1}{\delta^{1/7}}\, \delta_c^{8/7}\right) = \frac{k_c}{v_\infty}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{7}{72}\frac{1}{\delta^{1/7}}\, \delta_c^{8/7}\right) = \frac{k_c}{v_\infty}$$

for $S_c = 1 = \frac{\nu}{D_{AB}} = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}}$

$$\Rightarrow \delta = \delta_c$$

$$\Rightarrow \frac{d}{dx}\left(\frac{7}{72}\delta\right) = \frac{k_c}{v_\infty}$$

for turbulent flow. $\delta = \frac{0.371\,x}{Re_x^{1/5}}$

$$= \frac{0.371\,x}{\left(\frac{\rho \cdot x \cdot v_\infty}{\mu}\right)^{1/5}}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{7}{72} \cdot \frac{0.371}{\left(\frac{\rho \cdot v_\infty}{\mu}\right)^{1/5}} \cdot x^{4/5}\right) = \frac{k_c}{v_\infty}$$

$$\Rightarrow \frac{7}{72} \times \frac{0.371}{\left(\frac{\rho \cdot v_\infty}{\mu}\right)^{1/5}} \frac{4}{5} x^{-1/5} = \frac{k_c}{v_\infty}$$

$$\Rightarrow k_c = 0.0289 \frac{1}{\left(\frac{\rho \cdot x \cdot v_\infty}{\mu}\right)^{1/5}} v_\infty$$

$$= 0.0289 \frac{v_\infty}{Re_x^{1/5}}$$