25.2 Derive $\frac{\partial \rho_A}{\partial t} + (\vec{\nabla} \cdot \rho_A \vec{V}) - D_{AB} \nabla^2 \rho_A = r_A$ for a binary system,

(Sol) From differential eq'n of mass transfer

$$\vec{\nabla} \cdot \vec{n}_A - r_A + \frac{\partial \rho_A}{\partial t} = 0 \quad — ①$$

From Fick's eq'n:

$$\vec{n}_A = -\rho D_{AB} \vec{\nabla} \omega_A + \omega_A (\vec{n}_A + \vec{n}_B) \quad — ②$$

② substitutes into ①

$$\rho \times \frac{\rho_A \vec{v}_A + \rho_B \vec{v}_B}{\rho} = \rho \vec{V}$$

$$\vec{\nabla} \cdot (-\rho D_{AB} \vec{\nabla} \omega_A + \omega_A (\vec{n}_A + \vec{n}_B)) - r_A + \frac{\partial \rho_A}{\partial t} = 0$$

for constant ρ & D_{AB}

$$\Rightarrow -D_{AB} \vec{\nabla}^2 \rho_A + \vec{\nabla} \cdot \rho_A \vec{V} - r_A + \frac{\partial \rho_A}{\partial t} = 0$$

$$\Rightarrow \frac{\partial \rho_A}{\partial t} + (\vec{\nabla} \cdot \rho_A \vec{V}) - D_{AB} \vec{\nabla}^2 \rho_A = r_A$$