Midterm 2

Name_____

1. (8 pts) In using the von Kármán approximate method for analyzing the turbulent boundary layer over a flat plate, the following velocity and concentration profiles were assumed:

$$V_x = \alpha + \beta y \quad \text{and} \quad c_A = \eta + \xi y$$

The four constants, α, β, η, and ξ, are determined by the appropriate boundary conditions at the surface and at the outer edge of the hydrodynamic and concentration layers.

a. Please determine the resulting equations for velocity and concentration profiles.

b. Please determine the mass-transfer coefficient if the thickness of the turbulent boundary layer and the concentration boundary layer are given.

$$\delta = 0.8x \quad \delta c = 0.6x$$

2. (9 pts) Ozone gas (solute A) dissolved in water is used in wet cleaning processes. It is desired to produce a liquid water stream containing 3.0 gmole O_3/m^3 by a process that does not create any gas bubbles. One engineer's idea is shown in the figure. Liquid water containing 1.0 gmole O_3/m^3 enters a well-mixed tank at a volumetric flow rate 0.050 m^3/h. A pressurized gas mixture of O_3 diluted in N_2 is continuously added to the headspace of the tank at a total pressure of 1.5 atm. Both the liquid and gas inside the tank are assumed to be well mixed. The gas—liquid surface area inside the tank is 4.0 m^2. The process is maintained at 20°C. For a well-mixed, non-bubbled ozonation tank, the appropriate film mass-transfer coefficients for the liquid and gas films are $k_L = 3.0 \times 10^{-6}$ m/s and $k_g = 5.0 \times 10^{-3}$ gmole/s·m^2·atm, respectively. Equilibrium distribution data for O_3 gas dissolved in water at 20°C follows Henry's law, with H = 6×10^{-3} m^3 · atm/gmole.
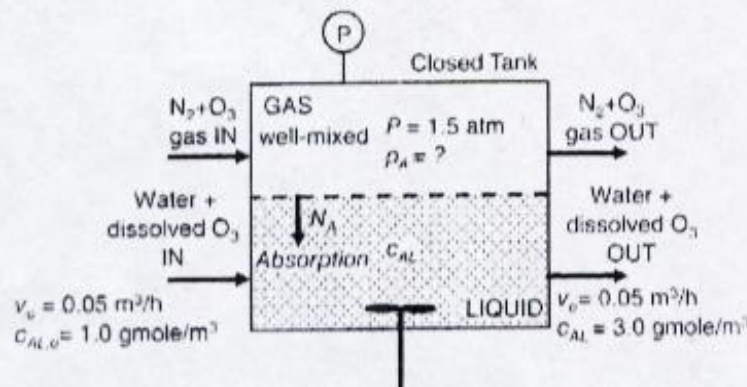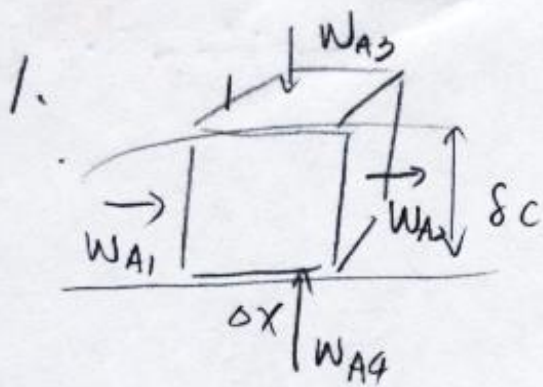
a. Is O_3 very soluble in water? Please explain the reason for that.

b. What is the overall mass-transfer coefficient K_G, based on the overall gas-phase driving forces?

c. What is the overall mass-transfer coefficient K_L, based on the overall liquid-phase driving forces?

d. For the process to operate as intended, what are the required partial pressure (p_A) and mole fraction (y_A) of ozone (O_3) in the gas phase inside the tank?

e. What is the total transfer rate of O_3, W_A?

f. Is the mass-transfer process gas film controlling, liquid film controlling?

3. (3 pts) Jasmone (species A) is obtained from the jasmine plant. A common method of manufacture is to extract the plant material in water, and then use benzene to concentrate the jasmone in a simple liquid-liquid extraction process. Jasmone is 170 times more soluble in benzene than in water, and so

$$C_A' = 170 C_A$$

Where C_A' is the concentration of jasmone in benzene, and C_A is the concentration of jasmone in water. In a proposed extraction unit, the benzene phase is well mixed with the film mass-transfer coefficient $k_L' = 3.5 \times 10^{-6}$ m/s. The aqueous phase is also well mixed with its film mass-transfer coefficient $k_L = 2.5 \times 10^{-5}$ m/s. Determine

a. The overall liquid mass-transfer coefficient, K_L', based on the benzene phase

b. The overall liquid transfer coefficient, K_L, based on the aqueous phase

c. The percent resistance to mass transfer encountered in the aqueous liquid film

1.

$$W_{A_1} + W_{A_3} + W_{A_4} = W_{A_J}$$

$$\int_0^{\delta_c} C_A \cdot U_x \cdot 1 \cdot dy \Big|_x + C_\infty \left[\frac{\partial}{\partial x} \int_0^{\delta_c} U_x \cdot dy \right] \Delta x$$

$$+ k_c \left(C_{AS} - C_{A\infty} \right) \cdot 1 \cdot \Delta x = \int_0^{\delta_c} C_A \cdot U_x \cdot 1 \cdot dy \Big|_{x + \Delta x}$$

$\Rightarrow$ dividing by Δx & Let $\Delta x \to 0$

$$\Rightarrow C_{A\infty} \left[\frac{d}{dx} \int_0^{\delta_c} U_x \, dy \right] + k_c \left(C_{AS} - C_{A\infty} \right) = \frac{d}{dx} \int_0^{\delta_c} C_A \cdot U_x \, dy$$

$$\Rightarrow \frac{d}{dx} \int_0^{\delta_c} (C_A - C_{A\infty}) U_x \, dy = k_c \left(C_{AS} - C_{A\infty} \right) \quad -\textcircled{1}$$

$\textcircled{+4}$

Assume $U_x = \alpha + \beta y$.

$$at \begin{cases} y = 0 \,, & U_x = 0 \qquad \Rightarrow 0 = \alpha \\ y = \delta \,, & U_x = U_\infty \quad \Rightarrow U_\infty = \beta \delta \Rightarrow \beta = \frac{U_\infty}{\delta} \end{cases}$$

$$\Rightarrow U_x = U_\infty \left(\frac{y}{\delta} \right) \quad -\textcircled{2}$$

$\textcircled{+1}$

Assume $C_A = \eta + \xi y$

at $\begin{cases} y=0, & C_A = C_{AS} \Rightarrow C_{AS} = \eta \\ y = \delta_c & C_A = C_{A\infty} \end{cases}$

$C_{A\infty} = C_{AS} + \xi \cdot \delta_c$

$\Rightarrow \xi = \dfrac{C_{A\infty} - C_{AS}}{\delta_c}$

$\Rightarrow C_A = C_{AS} + \left(C_{A\infty} - C_{AS}\right)\dfrac{y}{\delta_c}$ —③

(+1)

②, ③ substitute into ①

$$\frac{d}{dx}\int_0^{\delta_c} (C_{AS} - C_{A\infty})\left(1 - \frac{y}{\delta_c}\right) U_\infty \left(\frac{y}{\delta}\right) dy = k_c (C_{AS} - C_{A\infty})$$

(+1)

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_c} \left(\frac{y}{\delta} - \frac{y^2}{\delta_c \delta_c}\right) dy = \frac{k_c}{U_\infty}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta \delta_c}\right)\Bigg|_0^{\delta_c} = \frac{k_c}{U_\infty}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{\delta_c^2}{2\delta} - \frac{\delta_c^3}{3\delta_c \delta}\right) = \frac{k_c}{U_\infty}$$
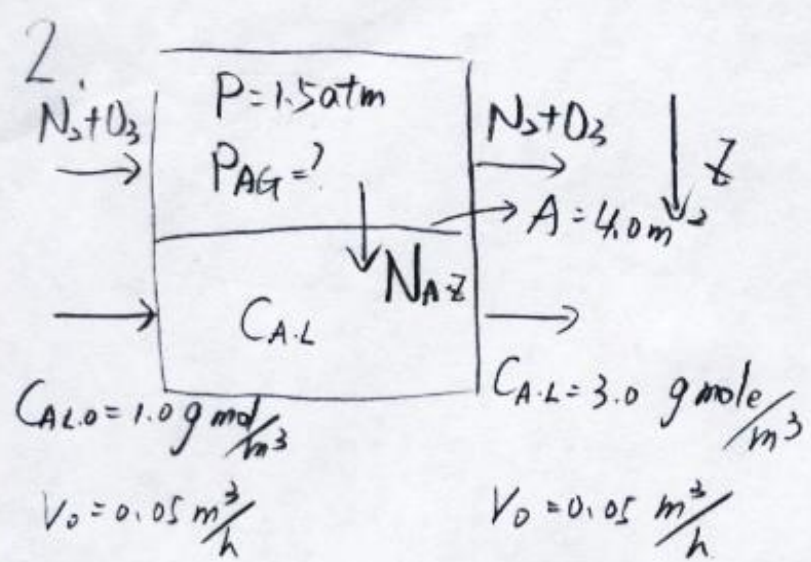
$$\Rightarrow \frac{d}{dx}\left(\frac{\delta_c^2}{2\delta} - \frac{\delta_c^2}{3\delta}\right) = \frac{k_c}{U_\infty}$$

③

$\Rightarrow \quad \dfrac{d}{dx}\left(\dfrac{(0.6x)^2}{2(0.8x)} - \dfrac{(0.6x)^2}{3\,0.8x}\right) = \dfrac{k_c}{U_\infty}$

$\Rightarrow \quad k_c = \left(\dfrac{0.36}{1.6} - \dfrac{0.6}{3}\right)U_\infty$

$\qquad\quad = 0.045\,U_\infty$

$\qquad\qquad\qquad\qquad$ +1

2.

Diagram labels: $N_3 + O_3 \rightarrow$; box with $P = 1.5 \, atm$, $P_{AG} = ?$, $\downarrow N_{Az}$, $C_{A \cdot L}$; $N_3 + O_3 \rightarrow$; $\rightarrow A = 4.0 \, m^2$ with $\bar{z}$; $\rightarrow$; $C_{A \cdot L} = 3.0 \, gmole/m^3$

$C_{AL.0} = 1.0 \, gmol/m^3$; $V_0 = 0.05 \, m^3/h$

$V_0 = 0.05 \, m^3/h$

a). $P_A^* = H C_A$,   Since H is small, O_3 is very soluble in water.   (+1)

b).

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{5 \times 10^{-3} \frac{gmol}{s \cdot m^2 \cdot atm}} + \frac{6 \times 10^{-3} \frac{atm \cdot m^3}{gmol}}{3 \times 10^{-6} \frac{m}{s}}$$

$$= 200 + 2000 \quad \frac{atm \cdot m^2 \cdot s}{gmol}$$

$$\Rightarrow K_G = \frac{1}{2200} = 4.55 \times 10^{-4} \quad \frac{gmol}{s \cdot m^2 \cdot atm} \quad (+1)$$

c)

$$\frac{1}{K_L} = \frac{1}{H \cdot k_G} + \frac{1}{k_L} = \frac{1}{6 \times 10^{-3} \cdot 5 \times 10^{-3} \frac{m}{s}} + \frac{1}{3 \times 10^{-6} \frac{m}{s}}$$

$$= \frac{1}{3 \times 10^{-5}} + \frac{1}{3 \times 10^{-6}}$$

$$= 3.33 \times 10^{4} + 3.33 \times 10^{5} = 3.66 \times 10^{4} \quad \frac{s}{m}$$

$$\Rightarrow K_L = \frac{1}{3.66 \times 10^{4}} = 2.73 \times 10^{-6} \quad \frac{m}{s} \quad (+1)$$

d) ⟲

for steady state

$$N_{A \cdot z} \cdot Area + C_{A \cdot L \cdot 0} \cdot \upsilon_0 - C_{A \cdot L} \cdot \upsilon_0 = 0$$

$$\Rightarrow N_{A \cdot z} = \frac{3 \frac{gmol}{m^3} \cdot 0.05 \frac{m^3}{h} - 1 \frac{gmol}{m^3} \cdot 0.05 \frac{m^3}{h}}{4.0 \ m^2}$$

$$= 0.025 \ \frac{gmol}{m^2 \cdot h} = 6.94 \times 10^{-6} \ \frac{gmol}{m^2 \cdot s} \quad ⊕$$

$$\underset{\shortparallel}{N_{A \cdot z}} = K_G \ (P_A - P_A^*) = K_G \ (P_A - H C_{A \cdot L})$$

$$6.94 \times 10^{-6} \ \frac{gmol}{m^2 \cdot s}$$

$$= 4.55 \times 10^{-4} \ (P_A - 6 \times 10^{-3} \cdot 3)$$

$$\frac{gmol}{s^2 \cdot m^2 \ atm} \qquad atm \qquad \frac{m^3 atm}{g \cdot mol} \cdot \frac{gmol}{m^3}$$

$$\Rightarrow P_A = \frac{6.94 \times 10^{-6}}{4.55 \times 10^{-4}} + 1.8 \times 10^{-2}$$

$$= 1.53 \times 10^{-2} + 1.8 \times 10^{-2}$$

$$= 3.33 \times 10^{-2} \ atm$$

$$y_A = \frac{3.33 \times 10^{-2}}{1.5} = 0.02 \quad ⊕$$

e.

$$W_A = N_A \cdot z \cdot \text{Area}$$

$$= 6.94 \times 10^{-6} \frac{g\,mol}{m^2 \cdot s} \cdot 4\,m^2$$

$$= 2.77 \times 10^{-5} \frac{g\,mol}{s} \quad \text{(+1)}$$

f.

$$\frac{\frac{1}{k_G}}{\frac{1}{K_G}} = \frac{200}{2200} = 0.09 \quad \Rightarrow \text{liqid film controlling}$$

(+1)

3.

$k_L' = 3.5 \times 10^{-6} \dfrac{kgmol}{m^2 \cdot s \cdot \left(\frac{kg \cdot mol}{m^3}\right)}$ $\overset{\text{benzene}}{C_{AL}^{①}} = 170 \; \overset{\text{aqueous}}{C_{A \cdot L}^{②}}$

$k_L = 2.5 \times 10^{-5} \dfrac{kgmol}{m^2 \cdot s \cdot \left(\frac{kgmol}{m^3}\right)}$

a.

$\dfrac{1}{K_L'} = \dfrac{1}{k_L'} + \dfrac{170}{k_L} = \dfrac{1}{3.5 \times 10^{-6} \frac{m}{s}} + \dfrac{170}{2.5 \times 10^{-5} \frac{m}{s}}$

$\Rightarrow K_L' = 1.41 \times 10^{-7} \; (\tfrac{m}{s})$ ⊕1

b.

$\dfrac{1}{K_L} = \dfrac{1}{170 \cdot k_L'} + \dfrac{1}{k_L} = \dfrac{1}{170 \cdot 3.5 \times 10^{-6} \frac{m}{s}} + \dfrac{1}{2.5 \times 10^{-5} \frac{m}{s}}$

$\Rightarrow K_L = 2.4 \times 10^{-5} \; (\tfrac{m}{s})$ ⊕1

c.

$\dfrac{\frac{1}{k_L}}{\frac{1}{K_L}} = \dfrac{K_L}{k_L} = \dfrac{2.4 \times 10^{-5} \; m/s}{2.5 \times 10^{-5} \; m/s} = 96\%$ ⊕1